Received: 11 December 2021 | Accepted: 30 April 2022

DOI: 10.1002/int.22912

RESEARCH ARTICLE

WILEY

A novel belief χ^2 divergence for multisource information fusion and its application in pattern classification

Lang Zhang | **Fuyuan Xiao**

School of Big Data and Software Engineering, Chongqing University, Chongqing, China

Correspondence
Fuyuan Xiao, School of Big Data and Software Engineering, Chongqing University, 401331 Chongqing, China.
Email: xiaofuyuan@cqu.edu.cn and doctorxiaofy@hotmail.com

Funding information
National Natural Science Foundation of China, Grant/Award Number: 62003280; Chongqing Talents: Exceptional Young Talents Project, Grant/Award Number: cstc2022ycjh-bgzxm0070

Abstract

Dempster–Shafer (D-S) evidence theory is invaluable in the domain of multisource information fusion for handing uncertainty problems. However, there may be counter-intuitive phenomenon when facing highly conflicting information. In this paper, a novel symmetric enhanced belief χ^2 divergence measure, called $SEB\chi^2$, is proposed to measure the discrepancy between basic probability assignments (BPAs). The $SEB\chi^2$ divergence consider the features of BPAs as the influence of both single-element subsets and multi-element subsets is taken into account. Furthermore, the $SEB\chi^2$ divergence is proven to be symmetric, nonnegative and nondegenerate, which are desirable properties for conflict management. Then, a new algorithm for multisource information fusion based on the $SEB\chi^2$ divergence measure is derived. Finally, an application for pattern classification is used to illustrate the superiority of the proposed $SEB\chi^2$ divergence measure-based fusion method over other existing well-known and recent related works with a better classification accuracy of 94.39%.

KEYWORDS

belief function, Dempster–Shafer theory, evidence conflict, multisource information fusion, pattern classification, symmetric enhanced belief χ^2 divergence, uncertainty

1 | INTRODUCTION

Multisource information fusion is an invaluable method used to combine the different information and generate a synthetic inference.[1] Therefore, multisource information fusion can be applied to a variety of areas, such as data deduplication,[2] intelligence analysis,[3] medical diagnosis,[4] picture fuzzy set,[5] reliability evaluation,[6,7] decision-making,[8–11] representation learning,[12] complex event processing,[13] and so on.[14–16] However, there is a challenge in multisource information fusion field that uncertain or even false results may be got when the data is interfered.[17] To address this problem, multisource information fusion uncertainty processing is necessary. Because it is a method to extract useful information. Then scholars are able to use this information to obtain reasonable results. In this case, a number of famous theories have been proposed for handing uncertain information,[18,19] such as fuzzy set theory,[20–22] N-soft,[23] d-number,[24] entropy-based,[25] and so on.[26–29]

In this paper, it is considered that D-S evidence theory provides an effective and flexible way to deal uncertainty problems; hence, this study focuses on uncertain multisource information fusion based on Dempster–Shafer (D-S) evidence theory.[30] Specially, the BPA can express the uncertainty by means of both singleton sets and multielement sets of an object.[31] In addition, with the law of associative and commutative, Dempster's rule of combination flexibly provides uncertainty reasoning to better support decision-making. Hence, D-S evidence theory is widely applied in the fields of target recognition,[32] output control,[33] pattern classification,[34] decision-making,[35] and so on.[36,37] Nevertheless, how to manage the highly conflicting evidence remains an open issue, since counterintuitive outcomes may be generated in such kind of situation. To solve this issue, a substantial amount of works have been done by modifying Dempster's combine rule or processing the body of evidence before information fusion.[38,39] In this paper, preprocessing the body evidence is taken into account to address problems of conflicting evidence. Studying some well-known works in the field of preprocessing of the body of evidence, it is noticed that scholars investigated the problem of conflicting evidence from different aspects, including distance, divergence measure, correlation coefficient, etc. Specifically, Han et al.[40] proposed a distance measure to represent the degree of dissimilarity between bodies of conflicting evidence.[41] Deng et al.[42] managed conflicting evidence based on correlation coefficient. Wang et al.[43] proposed a novel divergence measure to consider the discrepancy between evidence. More recently, Gao and Xiao[44] studied the conflict management problem based on a generalized χ^2 divergence. However, there is still a limitation.

Then main motivation of this study lies in the following points:

- In [44], the correlationship between BPAs was neglected, which means that it is supposed to take into consideration in $RB\chi^2$.
- It is important to improve and enhance the performance of the D-S evidence-based fusion system. Hence, a novel algorithm should be proposed to get the weight of each evidence through the degree of divergence.

However, there are still several challenges in this study:

- It is a challenge to consider features of BPAs for constructing a generalized χ^2 divergence to better measure conflict among evidence.
- It is difficult to build a more efficient information fusion algorithm to improve fusion performance.

To address above challenges, a novel symmetrical enhanced belief χ^2 divergence measure, called $SEB\chi^2$ divergence is proposed. Then, the discrepancy between two BPAs can be quantitatively figured out with $SEB\chi^2$ divergence. Besides, $SEB\chi^2$ divergence satisfies the properties of nonnegativeness, nondegeneracy and symmetry. Next, a new algorithm is derived for multisource information fusion by means of $SEB\chi^2$ divergence. Through studying several examples, it demonstrates that the $SEB\chi^2$ divergence measure can well distinguish the conflicts between BPAs. Furthermore, an application shows that $SEB\chi^2$ divergence-based multisource information fusion can effectively handle a real-word pattern classification problem.

Main contributions of this study lie in the following points:

- The proposed $SEB\chi^2$ divergence measure satisfies the properties of nonnegativeness, nondegeneracy and symmetry. In particular, the $SEB\chi^2$ divergence considers the discrepancy and correlationship between belief functions to measure the divergence in evidence theory.
- A new algorithm of D-S evidence-based multisource information fusion is derived relied on $SEB\chi^2$ divergence measure. Besides, this algorithm has good performance to manage conflicting evidence.
- An application in pattern classification is put forward with the new algorithm of $SEB\chi^2$ divergence-based multisource information fusion, which has more accurate results than the well-known works. In this case, the new algorithm can lead to more reasoning results.

This paper is organized as follows. Section 2 makes a review of the related works. In Section 3, the preliminaries of this study are briefly introduced. In Section 4, a novel divergence measure is proposed and some examples of comparative analysis are put forward. In Section 5, an algorithm of multisource information fusion is derived relied on the $SEB\chi^2$ divergence measure. In Section 6, an application of pattern classification is put forward to test the superiority of new algorithm of $SEB\chi^2$ divergence-based multisource information fusion by comparing with several well-known works, and some data is analyzed. Section 7 concludes this study.

2 | LITERATURE REVIEW

Multisource information fusion is a powerful technique that can fuse diverse information to obtain a comprehensive evaluation. Nevertheless, there is a challenge when facing the uncertain and conflicting information. Though DS evidence theory is an excellent method to handle uncertainty problems, it may lead to counter-intuitive results when facing highly conflict evidence. To manage the highly conflict evidence, scholars have proposed a variety of approaches, which can be divided into two main categories. One is to modify the combination rule of Dempster. Another is to modify the evidence before fusion them.

As for the modification of fusion rules, there are several well-known works. For example, Yager[45] thought the conflicting parts of evidence are invalid. So, Yager proposed a method to reassign the conflicting parts of evidence to unknown space. However, Yager's fusion method does not work well when there are more information sources. Moreover there is a serious problem that these modifications make the fusion rules lose the property of commutativity and associativity, which are incompatible with the basic rules. In addition, it is inappropriate to deliberately change the combination rule when there is a realistic conflict in evidence and lead to counter-intuitive results. So, the modification of evidence is taken into consideration.[46]

As for preprocessing the body of evidence, scholars try to alter the initial conflicting evidence to reasonable ones. Distance, divergence measure and correlation coefficient are important factors. Specifically, the divergence measure is an effective way to calculate the difference between evidence. For example, based on information volume of mass function, Gao et al.[47] proposed a generalized divergence to measure conflict between bodies of evidence. Wang et al.[43] proposed a new divergence measure to reflect the correlation of different kinds of subsets by taking into account the belief measure and plausibility measure of mass function. In addition, Chen and Cai[48] defined the modified Renyi-Belief divergence that integrates the characteristics of mass functions and can handle conflict by measuring the differences between mass functions. Xie et al.[49] improved multisensor fusion approach based on the cloud model and the belief Jensen-Shannon divergence. Moreover, Zhao et al.[50] measured divergence degree of basic probability assignment based on harmonic mean of Deng relative entropy. Recently, Gao and Xiao proposed a $RB\chi^2$ divergence to do conflict management. However, it is founded that $RB\chi^2$ has the limitation in considering the correlationship between BPAs. So, to consider the correlationship among evidence, a novel $SEB\chi^2$ divergence for multisource information fusion is proposed in this paper.

3 | PRELIMINARIES

This section briefly presents the fundamental concepts of D-S evidence theory and divergence measures of the classical $B\chi^2$.

3.1 | D-S evidence theory

D-S evidence theory is a generalization of typical probability theory,[30] which has a better performance in multisource information fusion with weaker conditional reasoning system.

Definition 1 (Framework of discernment). Let Θ be a set consisting of mutually exclusive and collectively exhaustive events,

$$\Theta = \{e_1, e_2, ..., e_n\}, \tag{1}$$

which indicates the discernment. Then, with Equation (1), its power set 2^Θ can be defined as follows:

$$2^\Theta = \{\emptyset, \{e_1\}, ..., \{e_n\}, \{e_1, e_2\}, ..., \{e_1, e_2, ..., e_h\}, ..., \Theta\}, \tag{2}$$

where $\emptyset$ indicates the empty set. Also, Song and Deng[51] proposed an entropic explanation of power set recently.

Definition 2 (Mass function). Based on the frame of discernment Θ, m as a belief function, also known as BPA, is a mapping from 2^Θ to $[0, 1]$. And with Equation (2), it can be defined as

$$m : 2^{\Theta} \rightarrow [0, 1]. \tag{3}$$

It also abides the rule of

$$\sum_{A \in 2^{\Theta}} m(A) = 1 \text{ and } m(\varnothing) = 0. \tag{4}$$

If $m(A) > 0$, A is a focal element.

As BPAs are able to model uncertainty, mass function has been thoroughly studied and derived, such as information volume of mass function,[52] complex evidential quantum dynamical model,[53] and so on.[54]

Definition 3 (Dempster's rule of combination). Let m_1 and m_2 be two BPAs. Dempster's combination rule is described in the form $m = m_1 \oplus m_2$:

$$m(A) = \begin{cases} \frac{1}{1-k} \sum_{B \cap C = A} m_1(B) m_2(C), & A \neq \varnothing, \\ 0, & A = \varnothing, \end{cases} \tag{5}$$

and

$$k = \sum_{B \cap C = \varnothing} m_1(B) m_2(C), \tag{6}$$

where B and C are focal elements. Besides, k is regarded as a coefficient to represent the degree of conflict between m_1 and m_2. Note that $0 \leq k < 1$ and the conflict becomes more obvious as k approaches one.

3.2 | Belief divergence measures

It remains an open issue for conflict management in the field of D-S evidence theory-based multisource information fusion. Some scholars improved the fusion model, while others did data processing before fusion. Here, this study focuses on the approach of preprocessing the body of evidence by utilizing divergence measure to calculate the discrepancy or degree of conflict among evidence, as divergence measure is widely used in decision-making.[55,56]

Definition 4 ($B\chi^2$ divergence). Let m_1 and m_2 be two independent BPAs. Then the $B\chi^2$ divergence between m_1 and m_2 can be defined as

$$B\chi^2(m_1, m_2) = \frac{1}{2}\left[\chi^2\left(m_1, \frac{m_1 + m_2}{2}\right) + \chi^2\left(m_2, \frac{m_1 + m_2}{2}\right)\right]. \tag{7}$$

$\chi^2(A,B)=\sum_{i=1}^{n}\frac{(a_i-b_i)^2}{b_i}$, where A and B be two probability distributions with $\sum_i^n a_i = \sum_i^n b_i = 1$. The a and b are the elements of A and B, respectively.

Note that the χ^2 divergence is unsymmetric and nonnegative. Then Gao and Xiao made an improvement and $B\chi^2$ divergence[44] is proposed. The $B\chi^2$ divergence has the following properties:

- $B\chi^2(m_1, m_2)$ is always symmetric.
- $B\chi^2(m_1, m_2)$ is bounded such that $0 \leq B\chi^2(m_1, m_2) \leq 1$.
- $B\chi^2(m_1, m_2) = 0$ if and only if $m_1 = m_2$.

4 | AN ENHANCED $B\chi^2$ DIVERGENCE MEASURE IN D-S EVIDENCE THEORY

This section starts by analyzing the correlation between BPAs. Next, a novel divergence measure named $EB\chi^2$ is devised. Then, the properties of $EB\chi^2$ divergence are proved. Last, numerical examples are given to show the performance of $EB\chi^2$ divergence.

4.1 | Correlation between the evidence

Based on Equation (7), the conflict degree between two BPAs can be measured to some extent. Nevertheless, there is a problem that $B\chi^2$ divergence lacks the correlation between BPAs. Here, an example is given to show this problem.

Example 1. With Equations (3) and (4), consider that three BPAs, m_1, m_2, and m_3, are based on the frame of discernment $\Theta = \{N_1, N_2, N_3, ..., N_8\}$:

$$\begin{aligned} m_1:\ & m_1(\{N_2, N_3, N_4\}) = 0.65,\ m_1(\{N_7\}) = 0.35; \\ m_2:\ & m_2(\{N_6, N_7, N_8\}) = 0.65,\ m_2(\{N_7\}) = 0.35; \\ m_3:\ & m_3(\{N_1, N_2, N_3, N_4, N_5\}) = 1. \end{aligned}$$

Obviously, m_1 and m_3 have the same element of $\{N_2, N_3, N_4\}$, while m_2 does not. So, the set $\{N_2, N_3, N_4\}$ of m_1 is much closer to the set $\{N_1, N_2, N_3, N_4, N_5\}$ of m_3 than the set $\{N_6, N_7, N_8\}$ of m_2 is. Hence, let ϱ be a function to measure the divergences between m_1, m_2 and m_3, respectively. Then the result is supposed to be

$$\varrho(m_1, m_3) < \varrho(m_2, m_3).$$

However, the following result is obtained based on $B\chi^2$ divergence:

$$B\chi^2(m_1, m_3) = B\chi^2(m_2, m_3) = 1.$$

So, it is significant to devise a measure that considers the correlation between the sets of BPAs, which means a more reasonable divergence measure should include both singleton sets and multielement sets. With this conjecture, the correlation coefficient is taken into account in the following part.

4.2 | An enhanced belief χ^2 divergence measure

In this section, χ^2 divergence and correlation coefficient are taken into account to obtain $EB\chi^2$ divergence. Then $SEB\chi^2$ divergence measure is proposed based on $EB\chi^2$ divergence.

Definition 5 ($EB\chi^2$ divergence). Let m_1 and m_2 be two BPAs based on the frame of discernment Θ, which includes n collectively exhaustive and mutually exclusive events. Then, $EB\chi^2$ divergence is defined by

$$D_{EB\chi^2}(m_1, m_2) = \sqrt{\left(\frac{m_1(\theta)}{\sqrt{m_2(\theta)}} - \sqrt{m_2(\theta)}\right)' \Psi \left(\frac{m_1(\theta)}{\sqrt{m_2(\theta)}} - \sqrt{m_2(\theta)}\right)}, \tag{8}$$

with $\theta \in \Theta$, and

$$\Psi(F_i, F_j) = \frac{2^{|F_i \cap F_j|} - 1}{2^{|F_i|} - 1} \cdot \frac{2^{|F_i \cap F_j|} - 1}{2^{|F_j|} - 1}, \tag{9}$$

where Ψ is the correlation coefficient;[57] F_i and F_j are the hypotheses of m_1 and m_2 $(i, j = 1, 2, ..., 2^{n-1})$; $|F_i|$ indicates the cardinality of F_i.

In Equation (9), if two sets are totally different, $|F_i \cap F_j| = 0$ and $\Psi(F_i, F_j) = 0$. On the contrary, if $F_i = F_j$, $|F_i \cap F_j| = |F_i| = |F_j|$ and $\Psi(F_i, F_j) = 1$. In this case, note that the correlation measure between two BPAs lies in their intersection.

Definition 6 (Symmetric enhanced belief divergence measure). Let Θ be the frame of discernment. And let m_1 and m_2 be two BPAs. Based on Equation (8), the symmetric enhanced belief χ^2 divergence measure $SEB\chi^2$ can be defined as

$$D_{SEB\chi^2}(m_1, m_2) = \frac{1}{2}\left[D_{EB\chi^2}\left(m_1, \frac{m_1 + m_2}{2}\right) + D_{EB\chi^2}\left(m_2, \frac{m_1 + m_2}{2}\right)\right]. \tag{10}$$

The $SEB\chi^2$ divergence is a generalized divergence that addresses the problem of ignoring correlation between sets. Moreover, the $SEB\chi^2$ has desirable properties which are analyzed as follows.

4.3 | Properties of $SEB\chi^2$ divergence measure

Properties: Given m_1, m_2 and m_3 as three BPAs on frame of discernment Θ. Then, three properties of $SEB\chi^2$ are shown as follows.

- *Nonnegativeness*: $D_{SEB\chi^2}(m_1, m_2) \geq 0$.

- *Nondegeneracy*: $D_{SEB\chi^2}(m_1, m_2) = 0$ if and only if $m_1 = m_2$.
- *Symmetry*: $D_{SEB\chi^2}(m_1, m_2) = D_{SEB\chi^2}(m_2, m_1)$.

Proof 1. Consider two BPAs m_1 and m_2. From

$$\Psi(F_i, F_j) = \frac{2^{|F_i \cap F_j|} - 1}{2^{|F_i|} - 1} \cdot \frac{2^{|F_i \cap F_j|} - 1}{2^{|F_j|} - 1},$$

know that $2^n - 1 \geq 0$, then $\Psi(F_i, F_j) \geq 0$. So, the result can be got

$$D_{EB\chi^2}(m_1, m_2) = \sqrt{\left(\frac{m_1(\theta)}{\sqrt{m_2(\theta)}} - \sqrt{m_2(\theta)}\right)' \Psi \left(\frac{m_1(\theta)}{\sqrt{m_2(\theta)}} - \sqrt{m_2(\theta)}\right)} \geq 0.$$

Therefore, $SEB\chi^2$ divergence is proved to be nonnegative with $D_{SEB\chi^2}(m_1, m_2) \geq 0$. □

Proof 2. Consider two BPAs m_1 and m_2 with $m_1 = m_2$. From

$$\Psi(F_i, F_j) = \frac{2^{|F_i \cap F_j|} - 1}{2^{|F_i|} - 1} \cdot \frac{2^{|F_i \cap F_j|} - 1}{2^{|F_j|} - 1},$$

note that $F_i = F_j$ as only when m_1 and m_2 have the same sets. Hence, $|F_i \cap F_j| = |F_i| = |F_j|$ and $\Psi(F_i, F_j) = 1$ can be obtained with two identical belief functions. In this case, $SEB\chi^2$ can be represented as

$$D_{EB\chi^2}(m_1, m_2) = \sqrt{\left(\frac{m_1(\theta)}{\sqrt{m_2(\theta)}} - \sqrt{m_2(\theta)}\right)' \left(\frac{m_1(\theta)}{\sqrt{m_2(\theta)}} - \sqrt{m_2(\theta)}\right)}.$$

So, $D_{EB\chi^2}(m_1, m_2) = 0$ is possible only if $m_1 = m_2$. Therefore, the $SEB\chi^2$ is proved to be nondegenerate. □

Proof 3. Consider $D_{SEB\chi^2}(m_1, m_2)$:

$$D_{SEB\chi^2}(m_1, m_2) = \frac{1}{2}\left[D_{EB\chi^2}\left(m_1, \frac{m_1 + m_2}{2}\right) + D_{EB\chi^2}\left(m_2, \frac{m_1 + m_2}{2}\right)\right].$$

Next, consider $D_{SEB\chi^2}(m_2, m_1)$:

$$D_{SEB\chi^2}(m_2, m_1) = \frac{1}{2}\left[D_{EB\chi^2}\left(m_2, \frac{m_2 + m_1}{2}\right) + D_{EB\chi^2}\left(m_1, \frac{m_2 + m_1}{2}\right)\right].$$

So, $D_{SEB\chi^2}(m_1, m_2) = D_{SEB\chi^2}(m_2, m_1)$ with the equation of $m_1 + m_2 = m_2 + m_1$. Hence, the property of symmetric is proven. □

4.4 | Performance of the proposed $SEB\chi^2$ divergence measure

By means of several examples, the performance of $SEB\chi^2$ divergence measure is illustrated in this section.

Let us back to *Example* 1. There are some limitations in certain cases with $B\chi^2$ divergence measure, as it overlooks the correlation between BPAs. Nevertheless, the following result will be get based on $SEB\chi^2$ divergence measure in Example 1:

$$D_{SEB\chi^2}(m_1, m_3) = 0.9;$$
$$D_{SEB\chi^2}(m_2, m_3) = 1.0.$$

It shows that $D_{SEB\chi^2}(m_1, m_3) < D_{SEB\chi^2}(m_2, m_3)$, which is in line with the intuition.

Example 2. Consider that two BPAs, m_1 and m_2, are based on the frame of discernment $\Theta = \{A, B, C, D, E\}$:

$$\begin{aligned} m_1: \ & m_1(\{A\}) = 0.55, m_1(\{B\}) = 0.15, m_1(\{C\}) = 0.10, \\ & m_1(\{D\}) = 0.10, m_1(\{E\}) = 0.10; \\ m_2: \ & m_2(\{A\}) = 0.55, m_2(\{B\}) = 0.15, m_2(\{C\}) = 0.10, \\ & m_2(\{D\}) = 0.10, m_2(\{E\}) = 0.10. \end{aligned}$$

Specially, m_1 and m_2 are the same and both of them consists only singleton sets. Based on $SEB\chi^2$ divergence, the result can be got

$$D_{SEB\chi^2}(m_1, m_2) = 0,$$

which illustrates that $SEB\chi^2$ divergence is measured to zero under the case of same singleton sets.

Example 3. Consider that two BPAs, m_1 and m_2, are based on the frame of discernment $\Theta = \{A, B, C, D, E\}$:

$$\begin{aligned} m_1: \ & m_1(\{A\}) = 0.55, m_1(\{B\}) = 0.15, m_1(\{C\}) = 0.10, \\ & m_1(\{D\}) = 0.10, m_1(\{A, B, C, D, E\}) = 0.10; \\ m_2: \ & m_2(\{A\}) = 0.55, m_2(\{B\}) = 0.15, m_2(\{C\}) = 0.10, \\ & m_2(\{D\}) = 0.10, m_2(\{A, B, C, D, E\}) = 0.10. \end{aligned}$$

Specially, m_1 and m_2 are the same and both of them consists singleton and multielement sets. Comparing with *Example* 2, the hypotheses of *Example* 3 include multielement sets.

Based on the $SEB\chi^2$ divergence, the result can be got

$$D_{SEB\chi^2}(m_1, m_2) = 0,$$

which illustrates that whether hypotheses consist of singleton sets or multielement sets, the $SEB\chi^2$ divergence measure is equal to 0 as long as two belief functions have the same element. In this case, the property of nondegeneracy is verified.

Example 4. Consider that two BPAs, m_1 and m_2, are based on the frame of discernment $\Theta = \{A, B, C, D, E\}$:

$$\begin{aligned} m_1: \; & m_1(\{A\}) = 0.55, m_1(\{B\}) = 0.15, m_1(\{C\}) = 0.10, \\ & m_1(\{D\}) = 0.10, m_1(\{A, B, C, D, E\}) = 0.10; \\ m_2: \; & m_2(\{A\}) = 0.60, m_2(\{B\}) = 0.10, m_2(\{C\}) = 0.10, \\ & m_2(\{D\}) = 0.10, m_2(\{A, B, C, D, E\}) = 0.10. \end{aligned}$$

This example shows that m_1 and m_2 are different. Specially, the divergence lies in the values of sets $\{A\}$ and $\{B\}$. In this case, the divergence of m_1 and m_2 can be calculated as

$$D_{SEB\chi^2}(m_1, m_2) = 0.2368.$$

For the same step, the divergence of m_2 and m_1 can be calculated as

$$D_{SEB\chi^2}(m_2, m_1) = 0.2368.$$

Therefore, the following result can be got:

$$D_{SEB\chi^2}(m_1, m_2) = D_{SEB\chi^2}(m_2, m_1),$$

which proves the property of symmetry.

Example 5. Consider two BPAs, m_1 and m_2. Besides, the belief value of $m_1(\{A, B, C\})$ constant spacing changes in the range of $[0.05, 0.95]$ as the coefficient φ. And a variable set Δ_t, is defined in both m_1 and m_2. It is shown in Table 1.

TABLE 1 Variable set Δ_t.

t	Δ_t
1	$\{A\}$
2	$\{A, B\}$
3	$\{A, B, C\}$
4	$\{A, B, C, D\}$
5	$\{A, B, C, D, E\}$
6	$\{A, B, C, D, E, F\}$
7	$\{A, B, C, D, E, F, G\}$
8	$\{A, B, C, D, E, F, G, H\}$
9	$\{A, B, C, D, E, F, G, H, I\}$
10	$\{A, B, C, D, E, F, G, H, I, J\}$

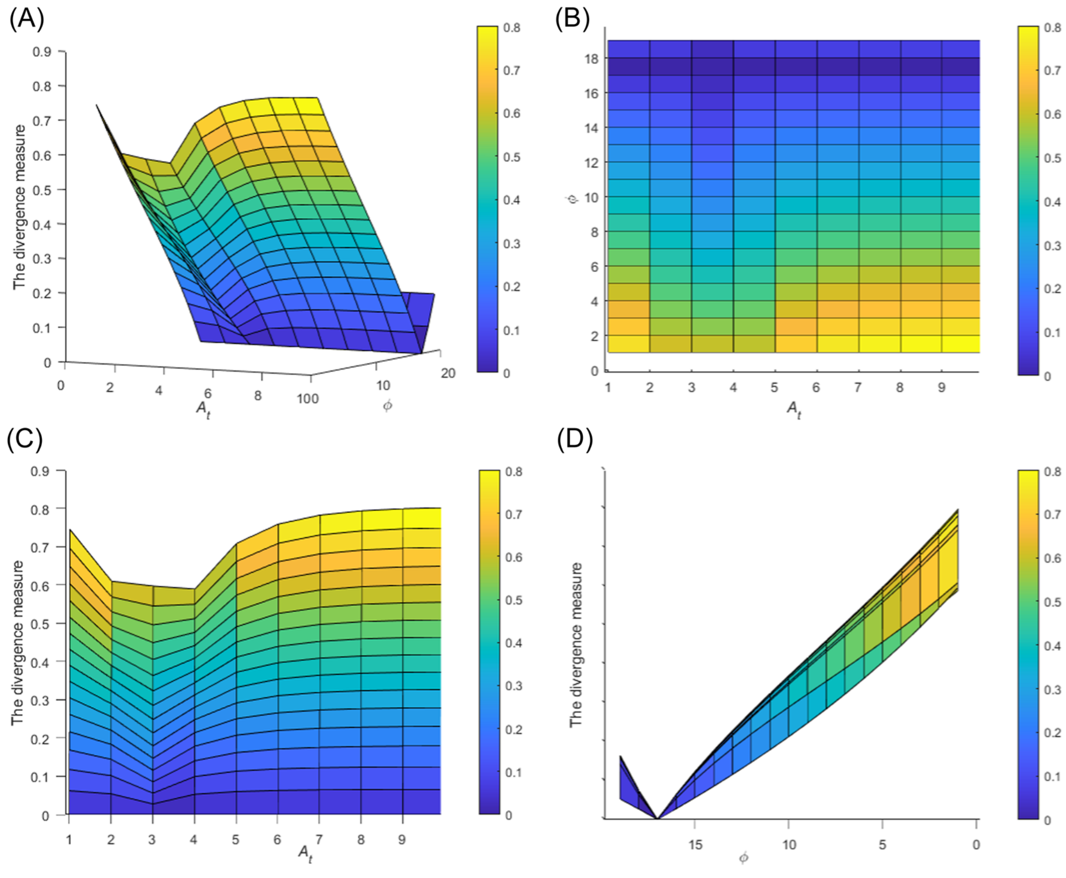

FIGURE 1 Behavior of the $SEB\chi^2$ divergence in Example 5. (A) Result for the $SEB\chi^2$ divergence measure, (B) variation of ϕ and A_t, (C) variation of the $SEB\chi^2$ divergence measure with varying A_t, and (D) variation of the $SEB\chi^2$ divergence measure with varying ϕ. [Color figure can be viewed at wileyonlinelibrary.com]

$$m_1: \quad m_1(\{A, B, C\}) = \varphi, m_1(\{\Delta_t\}) = 1 - \varphi;$$
$$m_2: \quad m_2(\{A, B, C\}) = 0.85, m_2(\{\Delta_t\}) = 0.15.$$

The behavior of $SEB\chi^2$ divergence measure is represented in Figure 1.

Specially in Figure 1C, with φ is fixed, note that there is a slight decline of $SEB\chi^2$ divergence measure when t increases from 1 to 3. Because with the addition of element B and C, two BPAs have more elements of intersection. Then, with the increase of t, the $SEB\chi^2$ divergence reaches the top and even surpasses the divergence value when $t = 1$. Figure 1C illustrates the significance of considering the intersection between subsets.

As shown in Figure 1B, m_2 gets closer to m_1 with φ increasing from 0.05 to 0.85 when t is fixed, as their same sets have higher value, which means the difference between two BPAs is smaller. Especially, when φ equals to 0.85, the $SEB\chi^2$ divergence measure equals to zero regardless the change of t.

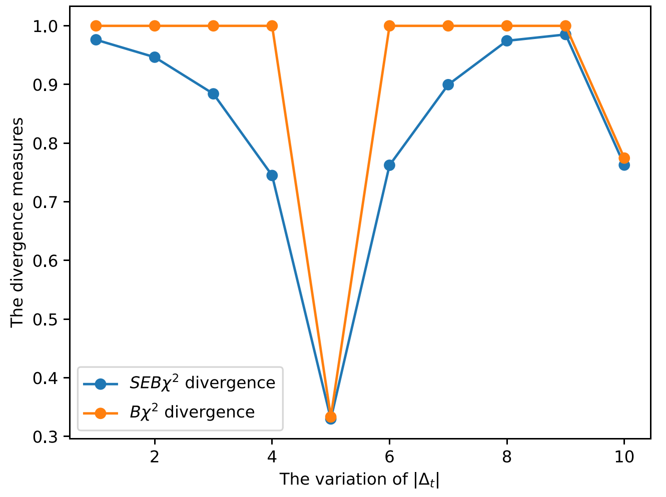

FIGURE 2 Comparison of the $B\chi^2$ and $SEB\chi^2$ divergence. [Color figure can be viewed at wileyonlinelibrary.com]

4.5 | Comparative analysis of the $B\chi^2$ and $SEB\chi^2$ divergence

In this section, to compare $B\chi^2$ and $SEB\chi^2$, an example is put forward.

Example 6. Consider that two BPAs, m_1 and m_2, are based on the frame of discernment Θ. Besides, the set of m_1 constant spacing changes in the rule of Δ_t defined in Table 1.

$$m_1: \quad m_1(\Theta) = 0.1, m_1(\{D, E, F\}) = 0.05, m_1(\{H\}) = 0.05, m_1(\Delta_t) = 0.8,$$
$$m_2: \quad m_2(\{A, B, C, D, E\}) = 1.$$

Figure 2 shows that the $SEB\chi^2$ divergence can well distinguish the discrepancy between BPAs. Specifically, as t increases from 1 to 5, $SEB\chi^2$ divergence measure tends to decrease. However, $B\chi^2$ divergence measure stays constant, as t increases from 1 to 4. In addition, as t increases from 5 to 10, $SEB\chi^2$ divergence measure tends to increase, while $B\chi^2$ divergence measure stays constant from 6 to 9. Note that the $B\chi^2$ is equal to 1 except only at $t = 5$ and 10. Based on this example, it is obvious that $SEB\chi^2$ has better performance than $B\chi^2$ in divergence measure as $SEB\chi^2$ divergence considers the correlationship between two BPAs.

5 | $SEB\chi^2$ DIVERGENCE-BASED MULTISOURCE INFORMATION FUSION

The $SEB\chi^2$ divergence measure is proposed to deal with the multisource information fusion with high-conflicting evidence. Based on $SEB\chi^2$ divergence measure, the procedure can be divided into following steps. First, the conflict measure of evidence can be acquired by means of $SEB\chi^2$ divergence measure. Then, the support degree can be got by average divergence. Next, the weight of each BPA is obtained based on support degree. Finally, the average weighted evidence is got and fused with Dempster's rule. To show the process more intuitively, a flow chart is put forward in Figure 3.

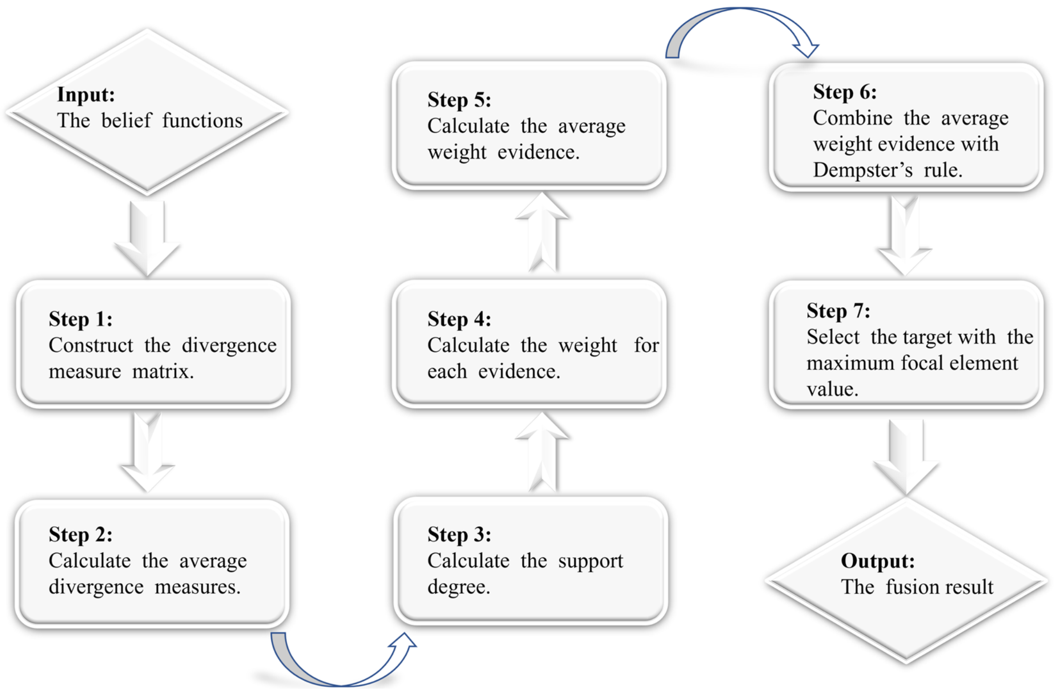

FIGURE 3 Flowchart of the $SEB\chi^2$ divergence-based multisource information fusion. [Color figure can be viewed at wileyonlinelibrary.com]

Step 1. Suppose that there are p BPAs m_i $(i = 1, 2, 3, ..., p)$. Then, the matrix DM of $SEB\chi^2$ divergence measure can be constructed as

$$DM = \begin{bmatrix} 0 & \dots & SEB\chi^2_{1i} & \dots & SEB\chi^2_{1p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ SEB\chi^2_{i1} & \dots & 0 & \dots & SEB\chi^2_{ip} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ SEB\chi^2_{p1} & \dots & SEB\chi^2_{pi} & \dots & 0 \end{bmatrix}. \tag{11}$$

Step 2. The average $SEB\chi^2$ divergence $Ave(m_i)$ of m_i can obtained as

$$Ave(m_i) = \frac{\sum_{j=1, i\neq j}^{p} SEB\chi^2_{ij}}{p-1}, 1 \leq j \leq p. \tag{12}$$

Know that the diagonal element is equal to zero, as the $D_{SEB\chi^2}(m_i, m_i) = 0$.

Step 3. The support degree sup for m_i can be calculated as

$$sup(m_i) = e^{-Ave(m_i)}. \tag{13}$$

Step 4. The weight $\omega(m_i)$ for each evidence can be calculated as

$$\omega(m_i) = \frac{sup(m_i)}{\sum_{i=1}^{p} sup(m_i)}. \tag{14}$$

Step 5. The average weighted average evidence $\bar{m}(E)$ can be calculated as

$$\bar{m}(E) = \sum_{i=1}^{p} \omega(m_i) \cdot m_i \ \ (E), E \subseteq \Theta. \tag{15}$$

Step 6. The average weighted evidence $\bar{m}$ is fused $p-1$ times by using Dempster's rule of combination with Equations (5) and (6):

$$F(\bar{m}) = \bar{m} \oplus \bar{m} \ \oplus \cdots \bar{m}, \tag{16}$$

where $F(\bar{m})$ are the combination results.

Step 7. Compare combination results in $F(\bar{m})$ and select the target with the maximum focal element value.

Algorithm 1: $SEB\chi^2$ divergence measure-based multisource information fusion

Input: A set of evidence $m = \{m_1, \ldots, m_i, \ldots, m_p\}$;
Output: Fusion result $F(\bar{m})$

for i=1;$i \leq p$ **do**
  for j=1;$j \leq p$ **do**
    Calculate the symmetric enhanced belief χ^2 divergence $SEB\chi^2_{ij}$ using Eq. (10);
  end
end
Construct the symmetric enhanced belief χ^2 divergence matrix DM using Eq. (11);
for i=1;$i \leq p$ **do**
  Compute the average divergence $Ave(m_i)$ of m_i using Eq. (12);
end
for i=1;$i \leq p$ **do**
  Generate the support degree $sup(m_i)$ of m_i using Eq. (13);
end
for i=1;$i \leq p$ **do**
  Calculate the weight $\omega(m_i)$ of m_i using Eq. (14);
end
for i=1;$i \leq p$ **do**
  Construct the average weighted evidence $\bar{m}(E)$ using Eq. (15);
end
for k=1;$k \leq p-1$ **do**
  Obtain the fusion result $F(\bar{m})$ using Eq. (16);
end
for i=1;$i \leq p$ **do**
  Select the target with the maximum focal element value;
end

Here, Algorithm 1 shows the pseudocode of the $SEB\chi^2$ divergence-based multisource information fusion algorithm.

6 | APPLICATION

Information fusion is widely used in data classification to identify the category that best fits.[58,59] Here, an application of flower-classification is put forward to demonstrate the effectiveness of the algorithm of $SEB\chi^2$ divergence-based multisource information fusion in following discussion.

6.1 | Application background

As for the data, Iris data sets that comes from UCI repository of the machine learning databases are chosen. In this data sets, note that they consist of Setosa (S), Versicolour (E), and Virginica (V) categories with 50 instances per class. Four attributes contribute to the Iris plants, denoted as petal length (pl), petal width (pw), sepal length (sl), and sepal width (sw).

6.2 | Data analysis based on Gaussian distribution

Note that the original data sets are just made up of feature data. So, the belief functions are calculated through following steps by Gaussian distribution.[60]

Step 1: Select training samples and test samples.

Here, 60% samples are randomly selected from the categories of S, E, and V of Iris data set, respectively, as training samples. These samples are used to build the Gaussian model on each attribute. In addition, the remaining 40% samples in each category are taken as test samples to calculate belief functions.

Step 2: Gauss model on each attribute is constructed.

Let X be the range of eigenvalues of a category on an attribute. The Gaussian membership function on each attribute can be defined as follows:

$$\mu(x) : X \to [0, 1], x \in X. \tag{17}$$

In following part, we show the step of $\mu(x)$.

For category i and attribute j, the mean $\bar{X}_{ij}$ and standard deviation σ_{ij} of samples are calculated separately. Then we get:

$$\mu_i^j(x) = exp\left[-\frac{(x - \bar{X}_{ij})^2}{2\sigma_{ij}^2}\right]. \tag{18}$$

In this case, based on Equations (17) and (18), the $\bar{X}$ and σ of each category and attribute can be calculated in Table 2.

Then, Gauss model can be drawn out in Figure 4.

TABLE 2 The mean and standard of Setosa, Versicolour, and Virginica with each attribute.

S	sl	sw	pl	pw
$\overline{X}$	5.057	3.467	1.457	0.233
σ	0.382	0.370	0.178	0.101
E	**sl**	**sw**	**pl**	**pw**
$\overline{X}$	5.870	2.770	4.213	1.317
σ	0.444	0.301	0.410	0.205
V	**sl**	**sw**	**pl**	**pw**
$\overline{X}$	6.597	3.017	5.573	2.093
σ	0.583	0.238	0.528	0.279

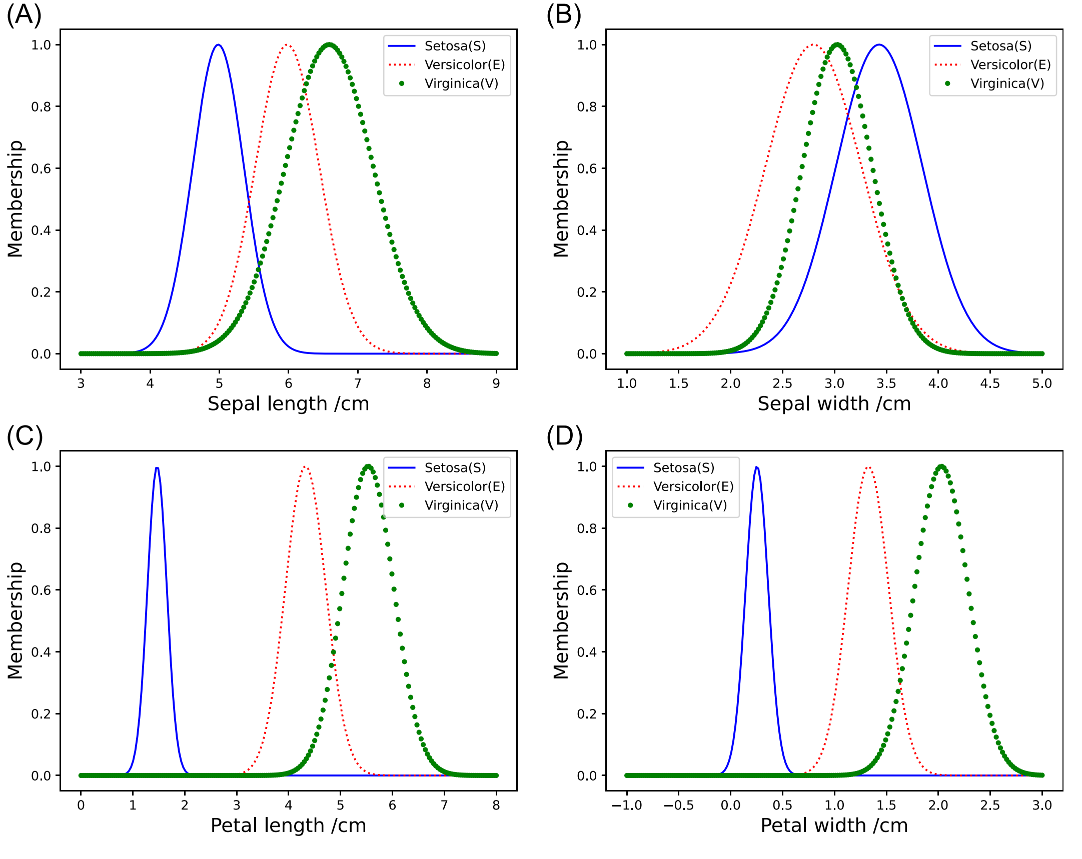

FIGURE 4 Gauss model of four attributes in three categories. (A) The membership with attribute sl, (B) the membership with attribute sw, (C) the membership with attribute pl, and (D) the membership with attribute pw. [Color figure can be viewed at wileyonlinelibrary.com]

Step 3: Match the sample sets with Gauss model. Suppose that there is a test sample as (5.1, 3.3, 1.7, 0.5). The belief functions of four attribute can be calculated by using generalized fuzzy numbers:

$$
\begin{aligned}
m_{sl} &: m_{sl}(S) = 0.7382, m_{sl}(S, E) = 0.1905, m_{sl}(S, E, V) = 0.0713;\\
m_{sw} &: m_{sw}(V) = 0.5034, m_{sw}(S, V) = 0.3264, m_{sw}(S, E, V) = 0.1701;\\
m_{pl} &: m_{pl}(S) = 0.9882, m_{pl}(S, E, V) = 0.0117;\\
m_{pw} &: m_{pw}(S) = 0.9944, m_{pw}(S, E, V) = 0.0055.
\end{aligned}
$$

Thus, we get four belief functions, namely, m_{sl}, m_{sw}, m_{pl}, m_{pw}.

6.3 | Implementation of the proposed method

In this section, the data before will be processed by using $SEB\chi^2$ divergence-based multisource information fusion.

Step 1. The $SEB\chi^2$ divergence measure matrix DM can be constructed as

$$
DM = \begin{bmatrix} 0 & 0.9877 & 0.3743 & 0.3897 \\ 0.9877 & 0 & 0.9998 & 1 \\ 0.3743 & 0.9998 & 0 & 0.0331 \\ 0.3897 & 1 & 0.0331 & 0 \end{bmatrix}.
$$

Step 2. The average divergence $Ave(m_i)$ of m_i can obtained as

$$
\begin{aligned}
Ave(m_1) &= 0.5839,\\
Ave(m_2) &= 0.9982,\\
Ave(m_3) &= 0.4691,\\
Ave(m_4) &= 0.4767.
\end{aligned}
$$

Step 3. The support degree sup for m_i can be calculated as

$$
\begin{aligned}
sup(m_1) &= 0.5577,\\
sup(m_2) &= 0.3685,\\
sup(m_3) &= 0.6256,\\
sup(m_4) &= 0.6208.
\end{aligned}
$$

Step 4. The weight $\omega(m_i)$ for each evidence can be calculated as

$$
\begin{aligned}
\omega(m_1) &= 0.2567,\\
\omega(m_2) &= 0.1696,\\
\omega(m_3) &= 0.2879,\\
\omega(m_4) &= 0.2858.
\end{aligned}
$$

Step 5. The average weighted evidence $\bar{m}$ can be calculated as

$$\bar{m}(S) = 0.7582,$$
$$\bar{m}(V) = 0.0854,$$
$$\bar{m}(S, V) = 0.0554,$$
$$\bar{m}(S, E) = 0.0489,$$
$$\bar{m}(S, E, V) = 0.0521.$$

Step 6. The average weighted evidence $\bar{m}$ is fused three times by using Dempster's rule of combination. And the results are presented as follows:

$$F(\bar{m}(S)) = 0.9979,$$
$$F(\bar{m}(V)) = 0.0018,$$
$$F(\bar{m}(S, V)) = 0.0002,$$
$$F(\bar{m}(S, E)) = 0.0001,$$
$$F(\bar{m}(S, E, V)) = 0.0000.$$

Step 7. From data above, we note that the accuracy of S reaches 99.79%, with this sample is selected from S, which means a correct forecast is produced.

6.4 | Comparison

Several classical and recent works are used in a pattern classification problem to make comparisons.

Table 3 and Figure 5 show the pattern classification accuracy. From Table 3, the overall accuracy based on $SEB\chi^2$ divergence measure reaches the value of 0.9439, which surpasses other methods we refer to.

From Figure 5A–C, we note that the main pattern classification accuracy influence factor is in the category of E and V. Table 3 shows those methods, except $RB\chi^2$ divergence measure-based fusion method,[44] perform well. Though there are accuracy losses in both categories E and V, it is obvious that the method with $SEB\chi^2$ divergence measure gets above average values in all categories. Compared with other methods that mentioned, $SEB\chi^2$ divergence measure-based fusion method achieves higher accuracy in E as 0.9394. Besides, compared with Deng et al.'s method[61] and Pan and Deng's method,[57] $SEB\chi^2$ divergence measure-based fusion method achieves higher accuracy in V as 0.8930.

TABLE 3 The pattern classification accuracy of different methods.

	Dempster[30]	Deng[61]	Murphy[62]	$RB\chi^2$ [44]	Pan and Deng[57]	$SEB\chi^2$
Overall	0.9207	0.9380	0.9378	0.8342	0.9422	**0.9439**
S	**1.0000**	**1.0000**	**1.0000**	0.9260	**1.0000**	0.9993
E	0.8400	0.9362	0.9106	0.7815	0.9348	**0.9394**
V	**0.9200**	0.8867	0.9060	0.7945	0.8919	0.8930

Note: The bold values illustrate the highest pattern classification accuracy on each data category and overall data based on different methods.

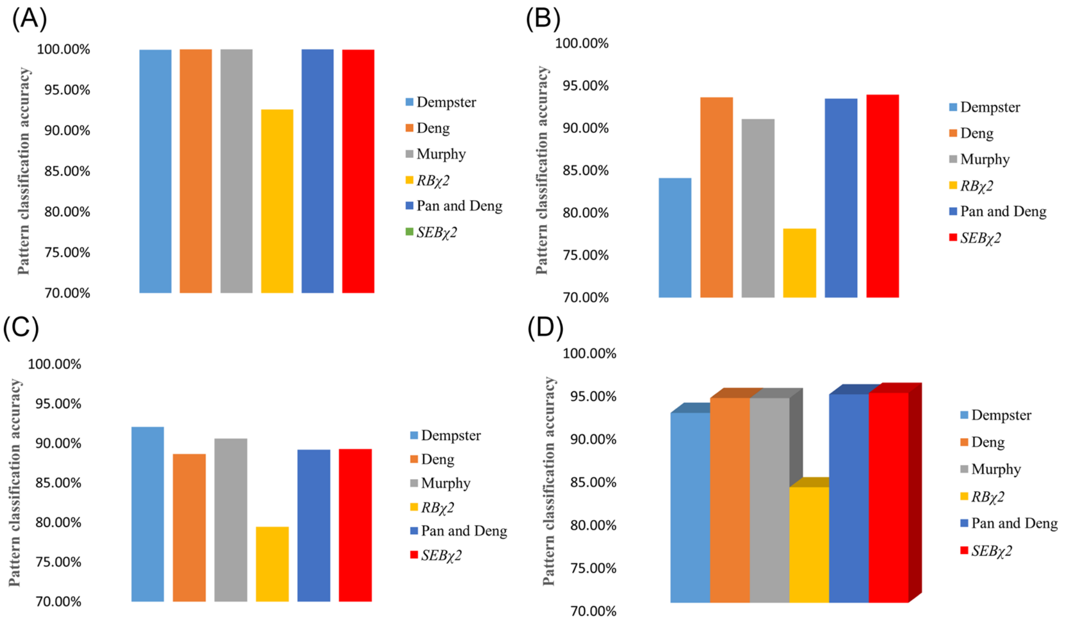

FIGURE 5 The pattern classification accuracy on each data category and overall data. (A) The accuracy on Setosa, (B) the accuracy on Versicolour, (C) the accuracy on Virginica, and (D) the accuracy on overall. [Color figure can be viewed at wileyonlinelibrary.com]

Hence, the above results illustrate the superiority of the algorithm of $SEB\chi^2$ divergence-based multisource information fusion in pattern classification problems. It can not only surpass well-known works (e.g., Dempster's,[30] Deng et al'.s[61] and Murphy's method[62]), but also have superior results compared with recent related works (e.g., $RB\chi^2$ divergence measure-based fusion method[44] and Pan and Deng's method[57]). $SEB\chi^2$ divergence further reflects the crucial role of correlationship between BPAs.

6.5 | Sensitivity analysis

In this section, an experiment based on the pattern classification problem above is carried out for the sensitivity analysis. This experiment is rendered to compare the robustness with those well-known works. At the beginning of experiment, we randomize the initial data of Iris to get test data sets of 100 times. Then, we use Dempster's method,[30] Deng et al.'s method,[61] Murphy's method,[62] $RB\chi^2$ divergence measure-based fusion method,[44] Pan and Deng's method[57] and the proposed method to fusion the BPAs.

In Figure 6D, through the overall pattern classification accuracy, we note that the proposed method performs better than Dempster's method,[30] Deng et al.'s method,[61] Murphy's method,[62] Pan and Deng's method[57] and $RB\chi^2$ divergence measure-based fusion method[44] as $SEB\chi^2$ divergence measure-based fusion method gets highest accuracy degree most of the time. Meanwhile, $SEB\chi^2$ divergence measure shows less volatile while its overall pattern classification accuracy floats among 90% to 95% in the experiment.

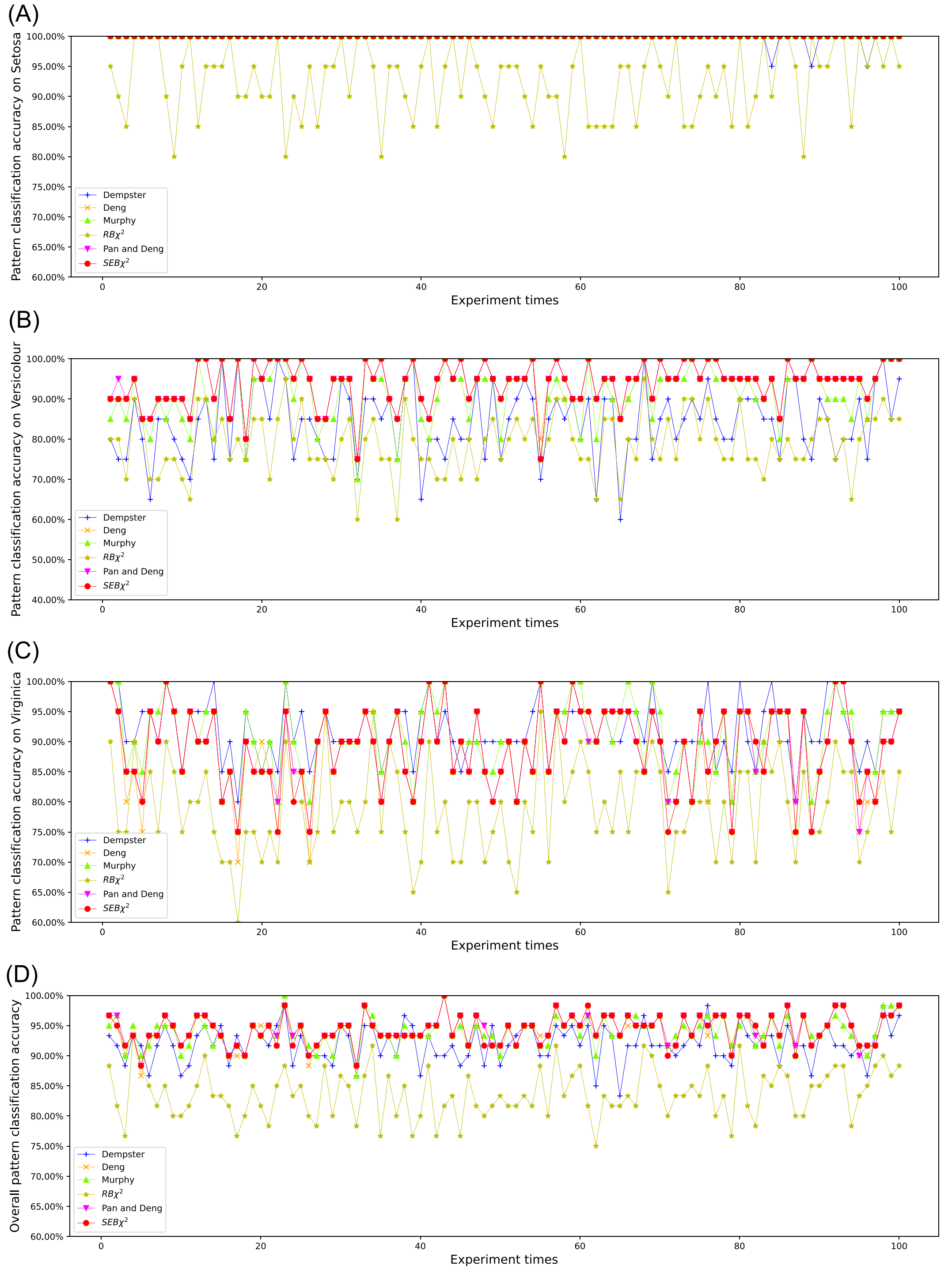

FIGURE 6 Result for the pattern classification accuracy by five existing multisource information fusion methods across 100 experiment times with Iris data. (A) The pattern classification accuracy of Setosa, (B) the pattern classification accuracy of Versicolour, (C) the pattern classification accuracy of Virginica, and (D) overall pattern classification accuracy. [Color figure can be viewed at wileyonlinelibrary.com]

In Figure 6A–C, they demonstrate the sensitivity through different categories. As shown in Figure 6A, we note that only Dempster's method[30] cannot reach the accuracy of 100% in the category of Setosa. From Figure 6B,C, although all the methods mentioned have fluctuation, the $SEB\chi^2$ divergence measure-based fusion method has a high degree of accuracy on most test sets.

In conclusion, the $SEB\chi^2$ divergence measure-based multisource information fusion outperforms Dempster's,[30] Murphy's,[62] Deng et al.'s,[61] Pan and Deng's method[57] and $RB\chi^2$ divergence measure-based fusion method[44] in the problem of pattern classification.

6.6 | Discussion

This paper proposes a novel $SEB\chi^2$ divergence measure. It firstly considers the features of BPAs as the influence of both single-element subset and multielement subsets is taken into consideration with $B\chi^2$ divergence measure. In this case, the research fills the gap that some classical well-known works neglect the correlationship with divergence between BPAs. At the same time, $SEB\chi^2$ divergence measure-based fusion algorithm gives a well-performed approach in the field of dealing with D-S theory-based uncertainty multisource information fusion problem.

From the application of pattern classification, it shows that $SEB\chi^2$ divergence measure leads to considerable results. Compared with classical well-known works (e.g., Dempster's method,[30] Deng et al's method[61] and Murphy's method[62]), $SEB\chi^2$ divergence measure-based information fusion illustrates the effectiveness in classification as it not only has high accuracy in classification but also good stability in sensitivity analysis. The effectiveness benefits from adequate consideration of the correlationship between BPAs in divergence measure. Consider recent related works (e.g., Pan and Deng's method[57] and $RB\chi^2$ divergence measure-based fusion method[44]), $SEB\chi^2$ divergence measure-based information fusion shows its superiority. Compared with $RB\chi^2$, $SEB\chi^2$ demonstrates that in the same basic framework of divergence, considering correlationship between BPAs can calculate the degree of divergence more valuable with higher result accurate. Besides, both Pan and Deng's method[57] and $SEB\chi^2$ take into account correlation between evidence. In addition, both of them have high accuracy in classification, which means that the outcomes of our proposed method are in line with recent works. In this case, an effective improvement in divergence is derived in this paper.

From the information above, compared with classical well-known works and recent related works, $SEB\chi^2$ divergence measure has the superiority for managing conflict evidence, as the correlationship between evidence in $B\chi^2$ divergence is taken into consideration. Based on several examples and application for pattern classification, the performance of the proposed method is verified.

7 | CONCLUSION

This study shed new light on a novel symmetric enhanced belief χ^2 divergence measure, named $SEB\chi^2$. Besides, some important properties of $SEB\chi^2$ divergence measure were proved, such as nonnegativeness, nondegeneracy and symmetry, which benefit to conflict measurement in evidence theory. Then main innovation point was that the correlation between the sets of belief functions were taken into consideration and be applied into classical $B\chi^2$ divergence. Also, the superiority and effectiveness of $SEB\chi^2$ divergence measure-based multisource information fusion was demonstrated with its highest accuracy rate. In summary, the $SEB\chi^2$ divergence measure provided an effective way in discrepancy distinguishing between BPAs. Moreover, this study provided a novel and well-performed solution to deal with the multisource information fusion problems. In future studies, the time consumption of the proposed method should be taken into account, and apply it to more complex and uncertain environments.

ACKNOWLEDGMENTS

The authors greatly appreciate the reviewers' suggestions and editor's encouragement. This study is supported by the National Natural Science Foundation of China (No. 62003280) and Chongqing Talents: Exceptional Young Talents Project (cstc2022ycjh-bgzxm0070).

CONFLICT OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

No data have been fabricated or manipulated (including images) to support the conclusions.

REFERENCES

1. Lai JW, Chang J, Ang L, Cheong KH. Multi-level information fusion to alleviate network congestion. *Inf Fusion*. 2020;63:248-255.
2. Li J, Li YK, Chen X, Lee PP, Lou W. A hybrid cloud approach for secure authorized deduplication. *IEEE Trans Parallel Distrib Syst*. 2014;26(5):1206-1216.
3. Fujita H, Gaeta A, Loia V, Orciuoli F. Hypotheses analysis and assessment in counter-terrorism activities: a method based on OWA and fuzzy probabilistic rough sets. *IEEE Trans Fuzzy Syst*. 2020;28:831-845.
4. Cao Z, Chuang C-H, King J-K, Lin C-T. Multi-channel EEG recordings during a sustained-attention driving task. *Sci. Data*. 2019;6:19. doi:10.1038/s41597-019-0027-4
5. Arya V, Kumar S. A novel TODIM-VIKOR approach based on entropy and Jensen-Tsalli divergence measure for picture fuzzy sets in a decision-making problem. *Int J Intell Syst*. 2020;35(12):2140-2180.
6. Meng D, Lv Z, Yang S, Wang H, Xie T, Wang Z. A time-varying mechanical structure reliability analysis method based on performance degradation. *Structures*. 2021;34:3247-3256.
7. Gao X, Su X, Qian H, Pan X. Dependence assessment in Human Reliability Analysis under uncertain and dynamic situations. *Nucl Eng Technol*. 2021;54:948–958. doi:10.1016/j.net.2021.09.045
8. Song Y, Fu Q, WangY-FmWang X. Divergence-based cross entropy and uncertainty measures of Atanassov's intuitionistic fuzzy sets with their application in decision making. *Appl Soft Comput*. 2019;84:105703.
9. Wu Z, Liao H. A consensus reaching process for large-scale group decision making with heterogeneous preference information. *Int J Intell Syst*. 2021;36:4560–4561. doi:10.1002/int.22469
10. Fu C, Chang W, Yang S. Multiple criteria group decision making based on group satisfaction. *Inf Sci*. 2020; 518:309-329.
11. Xie D, Xiao F, Pedrycz W. Information quality for intuitionistic fuzzy values with its application in decision making. *Eng Appl Artif Intell*. 2022;109:104568.
12. Fujita H, Ko Y-C. A heuristic representation learning based on evidential memberships: case study of UCI-SPECTF. *Int J Approx Reason*. 2020;120:125–137. doi:10.1016/j.ijar.2020.02.002
13. Xiao F. CaFtR: A fuzzy complex event processing method. *Int J Fuzzy Syst*. 2021;24:1098–1111. doi:10.1007/s40815-021-01118-6
14. Ni L, Chen Y-w, de Brujin O. Towards understanding socially influenced vaccination decision making: An integrated model of multiple criteria belief modelling and social network analysis. *Eur J Oper Res*. 2021; 293(1):276-289.
15. Li J, Li J, Chen X, Jia C, Lou W. Identity-based encryption with outsourced revocation in cloud computing. *IEEE Trans Comput*. 2013;64(2):425-437.

16. Verma R. Multiple attribute group decision-making based on order-α divergence and entropy measures under q-rung orthopair fuzzy environment. *Int J Intell Syst*. 2020;35(4):718-750.
17. Xiao F. Evidence combination based on prospect theory for multi-sensor data fusion. *ISA Trans*. 2020;106:253-261.
18. Baek Y, Yun U, Lin JC-W, Yoon E, Fujita H. Efficiently mining erasable stream patterns for intelligent systems over uncertain data. *Int J Intell Syst*. 2020;35(11):1699-1734.
19. Wei B, Xiao F, Fang F, Shi Y. Velocity-free event-triggered control for multiple Euler-Lagrange systems with communication time delays. *IEEE Trans Autom Control*. 2021;66(11):5599-5605.
20. Ye J, Zhan J, Ding W, Fujita H. A novel fuzzy rough set model with fuzzy neighborhood operators. *Inf Sci*. 2021;544:266-297.
21. Garg H. A new possibility degree measure for interval-valued q-rung orthopair fuzzy sets in decision-making. *Int J Intell Syst*. 2021;36(1):526-557.
22. Pedrycz W. *An Introduction to Computing with Fuzzy Sets—Analysis, Design, and Applications, Vol. 190 of Intelligent Systems Reference Library*. Springer; 2021. doi:10.1007/978-3-030-52800-3
23. Alcantud JCR, Feng F, Yager RR. An n-soft set approach to rough sets. *IEEE Trans Fuzzy Syst*. 2019;28(11):2996-3007.
24. Deng X, Jiang W. A total uncertainty measure for D numbers based on belief intervals. *Int J Intell Syst*. 2019;34(12):3302-3316.
25. Babajanyan S, Allahverdyan A, Cheong KH. Energy and entropy: path from game theory to statistical mechanics. *Phys Rev Res*. 2020;2(4):043055.
26. Wang Z, Wang C, Li X, Gao C, Li X, Zhu J. Evolutionary Markov dynamics for network community detection. *IEEE Trans Knowl Data Eng*. 2020:1206-1220. doi:10.1109/TKDE.2020.2997043
27. Wen T, Cheong KH. The fractal dimension of complex networks: a review. *Inf Fusion*. 2021;73:87-102.
28. Wang Z, Wang C, Gao C, Li X, Li X. An evolutionary autoencoder for dynamic community detection. *Sci China Inf Sci*. 2020;63(11):1-16.
29. Jia D, Wang X, Song Z, Romić I, Li X, Jusup M, Wang Z. Evolutionary dynamics drives role specialization in a community of players. *J R Soc Interface*. 2020;17(168):20200174.
30. Dempster AP. Upper and lower probabilities induced by a multivalued mapping. In: Yager RR, Liu L, eds. *Classic Works of the Dempster-Shafer Theory of Belief Functions*. Springer; 2008:57-72.
31. Deng Y. Uncertainty measure in evidence theory. *Sci China Inf Sci*. 2020;63(11):210201.
32. Huang C, Mi X, Kang B. Basic probability assignment to probability distribution function based on the Shapley value approach. *Int J Intell Syst*. 2021;36:4210-4236. doi:10.1002/int.22456
33. Chang L, Zhang L, Fu C, Chen Y-W. Transparent digital twin for output control using belief rule base. *IEEE Trans Cybern*. 2021. doi:10.1109/TCYB.2021.3063285
34. Liu Z, Liu Y, Dezert J, Cuzzolin F. Evidence combination based on credal belief redistribution for pattern classification. *IEEE Trans Fuzzy Syst*. 2020;28(4):618-631.
35. Liu P, Zhang X, Pedrycz W. A consensus model for hesitant fuzzy linguistic group decision-making in the framework of Dempster-Shafer evidence theory. *Knowl-Based Syst*. 2020;212:106559.
36. Xiong L, Su X, Qian H. Conflicting evidence combination from the perspective of networks. *Inf Sci*. 2021;580:408-418.
37. Tang S-W, Zhou Z-J, Hu C-H, Yang J-B, Cao Y. Perturbation analysis of evidential reasoning rule. *IEEE Trans Syst Man Cybern Syst*. 2021;51(8):4895-4910.
38. Li Y, Deng Y. Generalized ordered propositions fusion based on belief entropy. *Int J Comput Commun Control*. 2018;13(5):792-807.
39. Zhang W, Deng Y. Combining conflicting evidence using the DEMATEL method. *Soft Comput*. 2019;23(17):8207-8216.
40. Han D, Dezert J, Yang Y. Belief interval-based distance measures in the theory of belief functions. *IEEE Trans Syst Man Cybern Syst*. 2016;48(6):833-850.
41. Cheng C, Xiao F. A distance for belief functions of orderable set. *Pattern Recogn Lett*. 2021;145:165-170.
42. Deng J, Deng Y, Cheong KH. Combining conflicting evidence based on pearson correlation coefficient and weighted graph. *Int J Intell Syst*. 2021;36(12):7443-7460. doi:10.1002/int.22593

43. Wang H, Deng X, Jiang W, Geng J. A new belief divergence measure for Dempster-Shafer theory based on belief and plausibility function and its application in multi-source data fusion. *Eng Appl Artif Intell*. 2021;97:104030.
44. Gao X, Xiao F. A generalized χ^2 divergence for multisource information fusion and its application in fault diagnosis. *Int J Intell Syst*. 2021;37(1):5-29.
45. Yager RR. On the Dempster-Shafer framework and new combination rules. *Inf Sci*. 1987;41(2):93-137.
46. Song X, Xiao F. Combining time-series evidence: A complex network model based on a visibility graph and belief entropy. *Appl Intell*. 2022. doi:10.1007/s10489-021-02956-5
47. Gao X, Pan L, Deng Y. A generalized divergence of information volume and its applications. *Eng App Artif Intell*. 2022;108:104584.
48. Chen Z, Cai R. A novel divergence measure of mass function for conflict management. *Int J Intell Syst*. 2021;37:3709-3735. doi:10.1002/int.22741
49. Xie S, Chen Y, Dong S, Zhang G. Risk assessment of an oil depot using the improved multi-sensor fusion approach based on the cloud model and the belief Jensen-Shannon divergence. *J Loss Prev Process Ind*. 2020;67:104214.
50. Zhao K, Sun R, Li L, Hou M, Yuan G, Sun R. An optimal evidential data fusion algorithm based on the new divergence measure of basic probability assignment. *Soft Comput*. 2021;25(17):11449-11457.
51. Song Y, Deng Y. Entropic explanation of power set. *Int J Comput Commun Control*. 2021;16(4):4413.
52. Deng Y. Information volume of mass function. *Int J Comput Commun Control*. 2020;15(6):3983.
53. Xiao F. CEQD: A complex mass function to predict interference effects. *IEEE Trans Cybern*. 2021. doi:10.1109/TCYB.2020.3040770
54. Chen L, Deng Y, Cheong KH. Probability transformation of mass function: a weighted network method based on the ordered visibility graph. *Eng Appl Artif Intell*. 2021;105:104438.
55. Wang Z, Xiao F, Ding W. Interval-valued intuitionistic fuzzy Jenson-Shannon divergence and its application in multi-attribute decision making. *Appl Intell*. 2022. doi:10.1007/s10489-022-03347-0
56. Khan MJ, Alcantud JCR, Kumam P, Kumam W, Al-Kenani AN. An axiomatically supported divergence measures for q-rung orthopair fuzzy sets. *Int J Intell Syst*. 2021;36(10):6133-6155.
57. Pan L, Deng Y. An association coefficient of a belief function and its application in a target recognition system. *Int J Intell Syst*. 2020;35(1):85-104.
58. Jia D, Guo H, Song Z, Shi L, Deng X, Perc M, Wang Z. Local and global stimuli in reinforcement learning. *New J Phys*. 2021;23(8):083020.
59. Wang Z, Li Z, Wang R, Nie F, Li X. Large graph clustering with simultaneous spectral embedding and discretization. *IEEE Trans Pattern Anal Mach Intell*. 2021;43(12):4426-4440.
60. Jiang W, Deng X. *D-S Evidence Theory Information Modeling and Application*. Science Press; 2018.
61. Deng Y, Shi W, Zhu Z, Liu Q, Combining belief functions based on distance of evidence. *Decis Support Syst*. 2004;38(3):489-493.
62. Murphy CK. Combining belief functions when evidence conflicts. *Decis Support Syst*. 2000;29(1):1-9.

How to cite this article: Zhang L, Xiao F. A novel belief χ^2 divergence for multisource information fusion and its application in pattern classification. *Int J Intell Syst*. 2022;1-24. doi:10.1002/int.22912